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SUB -TRIDENT FORM THROUGH FUZZY SUB -TRIANGULAR FORM

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ABSTRACT

This paper deals with the new concept to find Shortest Path and the Optimum Solution with the help of Fuzzy Numbers. Here the Fuzzy Sub-Triangular Form is obtained from the Pascal's Triangle Graded Mean along with the help of fuzzy numbers and this form is again converted to Sub-Trident Form. The Minimum value of Sub-Trident Form gives the shortest Path and also the Optimum Solution with suitable numerical example.

KEYWORDS: Graded Mean, Fuzzy Numbers, Optimum Solution, Pascal's Triangle, and Sub-Trident Form.

INTRODUCTION

One of the most important problem in transportation network is the shortest path problem. The distance of shortest route is calculated using this shortest path problem from source node to the destination node. Dubois and Prade introduced the fuzzy shortest path problem with the help of Floyd's algorithm in the year 1980 [2].Later in the year 1994, Okada and Gen [3] introduced the same problem with the help of Dijkstra's algorithm. Lotfi.A.Zadeh introduced the fuzzy set theory in the year 1965[1].Then Chen and Hsieh introduced the Fuzzy Graded Mean Integration Representation [6] and [7]. After that the representation and application of fuzzy number is given by S.Hilpern in the year 1997 [4].In the year 2000, Okada and Soper concentrated on shortest path problem on a network in which a fuzzy number, instead of a real number is assigned to each arc length [5]. In this proposed method the Sub-Trident Form through Fuzzy Sub-Trident Form by using trapezoidal fuzzy numbers with the help of Pascal's Triangle Graded Mean which gives the Shortest Path and the Optimal Solution. Here this paper consists of Sections: Basic definitions and notations in the first section, the proposed method in the second section, the Working Rule or the Algorithm in the third section, identifying the shortest path and obtaining the optimal solution by giving suitable numerical example in the fourth section and finally the fifth section gives the conclusion based on our study.

PRELIMINARIES

In this section, some basic definition of fuzzy set theory and fuzzy number are discussed [11].

Definition :1 A fuzzy set $\tilde{A}$ in X is characterized by a membership function $\mu_{\tilde{A}}(x)$ represents grade of membership of $x \in \mu_{\tilde{A}}(x)$. More general representation for a fuzzy set is given by

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) / x \in X \right\}.$$

Definition :2 The $\alpha - cut$ of a fuzzy set $\tilde{A}$ of the Universe of discourse X is defined as

$\tilde{A} = \{ x \in X / \mu_A(x) \geq \alpha \}$, where $\alpha \in [0,1]$.

Definition 2.3. A fuzzy set $\tilde{A}$ defined on the set of real numbers $\Re$ is said to be a fuzzy number if its membership function $\tilde{A}:\Re \rightarrow [0,1]$ has the following characteristics.

a) $\tilde{A}$ is convex if
$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\} \forall x_1, x_2 \in X, \lambda \in [0,1]$$

b) $\tilde{A}$ is normal if there exists an $x \in \Re$ such that if $\max \mu_{\tilde{A}}(x) = 1.$

c) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Representation of Generalized (Trapezoidal) Fuzzy Number

In general, a generalized fuzzy number A is described at any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions:

- μ_A is a continuous mapping from R to [0,1],
- μ_A (x)=0, $-\infty \prec x \leq c,$
- $\mu_A(x) = L(x)$ is strictly increasing on [c,a]
- $\mu_A(x) = w, a \leq x \leq b,$
- $\mu_A(x) = R(x)$ is strictly decreasing on [b,d],
- $\mu_A(x) = 0, d \leq x \prec \infty$ Where $0 \prec w \leq 1$ and a, b, c and d are real numbers.

We denote this type of generalized fuzzy number as $A = (c,a,b,d;w)_{LR}$. When w=1, we denote this type of generalized fuzzy number as $A = (c,a,b,d)_{LR}$. When L(x) and R(x) are straight line, then A is Trapezoidal fuzzy number, we denote it as (c, a, b, d).

Graded Mean Integration Representation

In 1998, Chen and Hsieh [6] and [7] proposed graded mean integration representation for representing generalized fuzzy number. Suppose L^{-1}, R^{-1} are inverse functions of L and R respectively, and the graded mean h-level value of generalized fuzzy number $A = (c,a,b,d;w)_{LR}$ is $h[L^{-1}(h) + R^{-1}(h)]/2$. Then the graded mean integration representation of generalized fuzzy number based on the integral value of graded mean h-level is

$$P(A) = \frac{\int_0^w h\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right)dh}{\int_0^w h\,dh} = \frac{\mathrm{c + 2a + 2b + d}}{6}$$

Where h is between 0 and w, 0<w ≤1;

Pascal's Triangle Graded Mean Approach

The Graded Mean Integration Representation for generalized fuzzy number by Chen and Hsieh [6] - [8].Later Sk.Kadhar Babu and B.Rajesh Anand introduces Pascal's Triangle Graded Mean in Statistical Optimization [10].But the present approach is a very simple was of analyzing fuzzy variables to get the optimum shortest path. This

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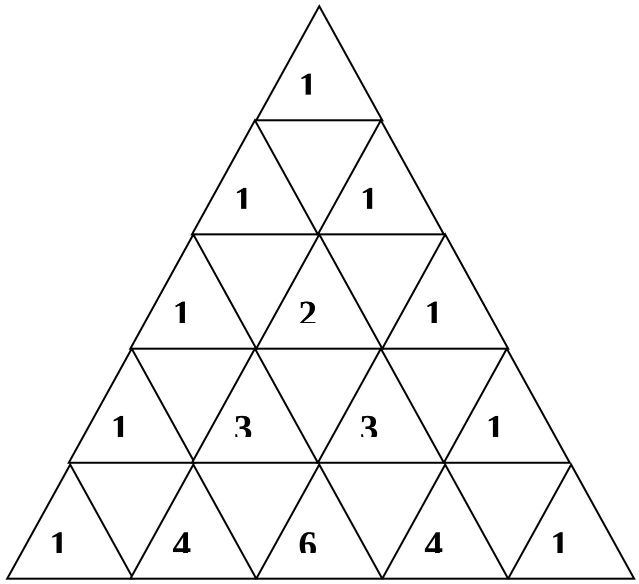
procedure is taken from the following Pascal's triangle. We take the coefficients of fuzzy variables as Pascal's triangle numbers. Then we just add and divide by the total of Pascal's number and we call it as Pascal's Triangle Graded Mean Approach.

Figure:1



Pascal's Triangle

The following are the Pascal's triangular approach:

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers then we can take the coefficient of fuzzy numbers from Pascal's triangles and apply the approach we get the following formula:

$$P(A) = \frac{a_1 + 3a_2 + 3a_3 + a_4}{8}; P(B) = \frac{b_1 + 3b_2 + 3b_3 + b_4}{8};$$

The coefficients of a_1, a_2, a_3, a_4 and b_1, b_2, b_3, b_4 are 1, 3, 3, 1.This approach can be extended fir n-dimensional Pascal's Triangular fuzzy order also.

PROPOSED METHOD

Fuzzy Sub-Triangular Form of Pascal's Triangle

A. Triangular Fuzzy Numbers:

The Pascal's Triangle for Triangular Fuzzy Number is given in figure: 2 and the Sub -Triangles for Triangular Fuzzy Number is given in Figure: 3 (*Set: I*) as follows:

Figure:2

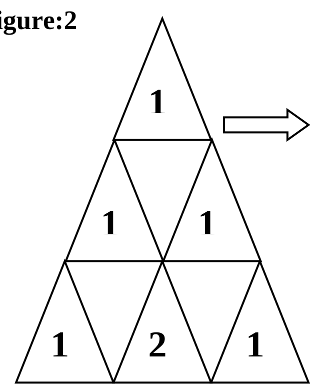
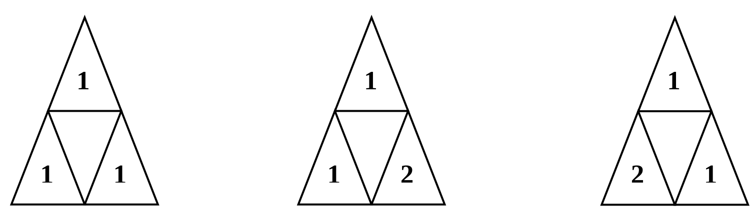


Figure: 3



Sub Triangle: (i) **Sub-Triangle: (ii)** **Sub-Triangle: (iii)**

Set: I

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P1= $P(A) = \frac{a_1 + a_2}{2}$, q1= $P(B) = \frac{a_1 + a_2}{2}$, r1= $P(C) = \frac{a_1 + a_2}{2}$.

P2= $P(A) = \frac{a_1 + a_2}{2}$, q2= $P(B) = \frac{a_1 + 2a_2}{3}$, r2= $P(C) = \frac{2a_1 + a_2}{3}$.

P3= $P(A) = \frac{a_1 + 2a_2}{3}$, q3= $P(B) = \frac{2a_1 + a_2}{3}$, r3= $P(C) = \frac{a_1 + a_2}{2}$.

The Fuzzy Sub-Triangular Form for Triangular Fuzzy Number is given by

$$FST_f = (p_p, q_q, r_r), \text{ where } p_p = \frac{p_1 + p_2 + p_3}{3}, q_q = \frac{q_1 + q_2 + q_3}{3}, r_r = \frac{r_1 + r_2 + r_3}{3}$$

B. Trapezoidal Fuzzy Numbers:

The Pascal's Triangle for Trapezoidal Fuzzy Number is given in figure: 4 and the Sub -Triangles for Trapezoidal Fuzzy Number is given in Figure: 5(*Set: II*) as follows:

Figure: 4

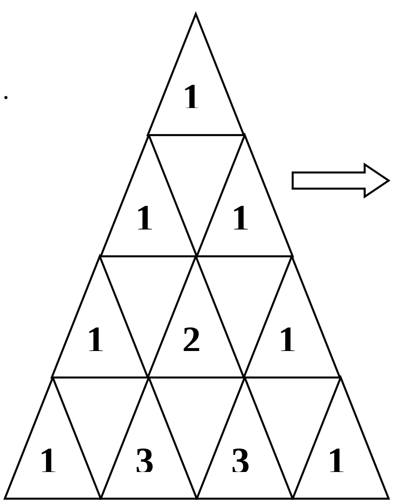
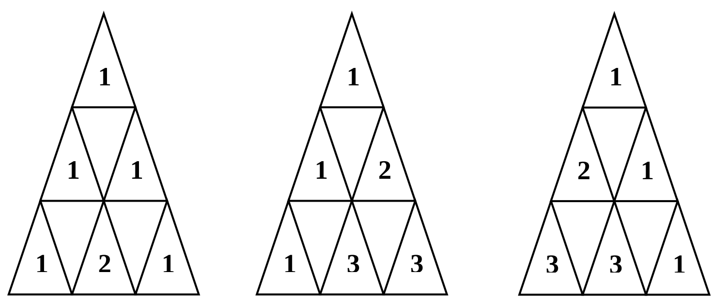


Figure: 5



Sub Triangle: (i) **Sub-Triangle: (ii)** **Sub-Triangle: (iii)**

Set: II

P1 = $P(A) = \frac{a_1 + a_2 + a_3}{3}$, q1= $P(B) = \frac{a_1 + 2a_2 + a_3}{4}$, r1= $P(C) = \frac{a_1 + a_2 + a_3}{3}$.

P2 = $P(A) = \frac{a_1 + a_2 + a_3}{3}$, q2= $P(B) = \frac{a_1 + 3a_2 + 3a_3}{7}$, r2= $P(C) = \frac{3a_1 + 2a_2 + a_3}{6}$.

P3 = $P(A) = \frac{a_1 + 2a_2 + 3a_3}{6}$, q3= $P(B) = \frac{3a_1 + 3a_2 + a_3}{7}$, r3= $P(C) = \frac{a_1 + a_2 + a_3}{3}$.

The Fuzzy Sub-Triangular Form for Trapezoidal Fuzzy Number is given by

$$FST_f = (p_p, q_q, r_r), \text{ where } p_p = \frac{p_1 + p_2 + p_3}{3}, q_q = \frac{q_1 + q_2 + q_3}{3}, r_r = \frac{r_1 + r_2 + r_3}{3}$$

C. Pentagonal Fuzzy Numbers:

The Pascal's Triangle for Pentagonal Fuzzy Number is given in figure: 6 and the Sub -Triangles for Pentagonal Fuzzy Number is given in Figure: 7(*Set: III*) as follows:

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Figure: 6

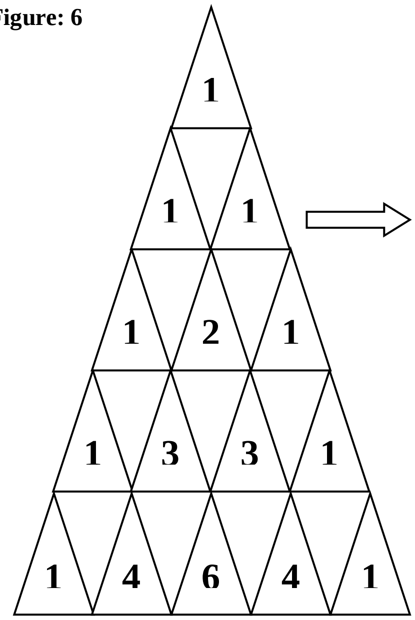
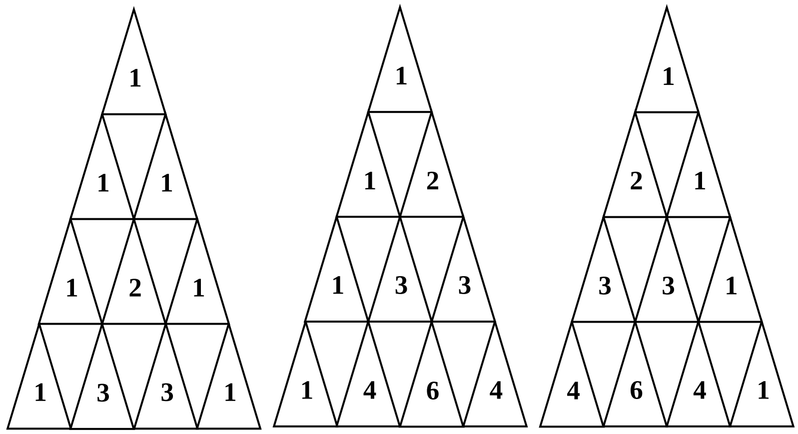


Figure: 7



Sub Triangle: (i) Sub-Triangle: (ii) Sub-Triangle: (iii)

Set: III

$P_1 = P(A) = \dfrac{a_1 + a_2 + a_3 + a_4}{4}$, $q_1 = P(B) = \dfrac{a_1 + 3a_2 + 3a_3 + a_4}{8}$, $r_1 = P(C) = \dfrac{a_1 + a_2 + a_3 + a_4}{4}$.

$P_2 = P(A) = \dfrac{a_1 + a_2 + a_3 + a_4}{4}$, $q_2 = P(B) = \dfrac{a_1 + 4a_2 + 6a_3 + 4a_4}{15}$, $r_2 = P(C) = \dfrac{4a_1 + 3a_2 + 2a_3 + a_4}{10}$.

$P_3 = P(A) = \dfrac{a_1 + 2a_2 + 3a_3 + 4a_4}{10}$, $q_3 = P(B) = \dfrac{4a_1 + 6a_2 + 4a_3 + a_4}{15}$, $r_3 = P(C) = \dfrac{a_1 + a_2 + a_3 + a_4}{4}$.

The Fuzzy Sub-Triangular Form for Pentagonal Fuzzy Number is given by

$FST_f = (p_p, q_q, r_r)$, where $p_p = \dfrac{p_1 + p_2 + p_3}{3}, q_q = \dfrac{q_1 + q_2 + q_3}{3}, r_r = \dfrac{r_1 + r_2 + r_3}{3}$

Similarly we can extend this to different fuzzy numbers.

Sub-Trident Form

The Sub-Trident Form of Fuzzy Number is given by $ST_{ri} = \dfrac{1}{3}\left[p_p^{1/3} + q_q^{1/3} + r_r^{1/3}\right]$, where

p_p, q_q, r_r are the Graded Means of the Pascal's Triangle from the Fuzzy Triangular Form.

ALGORITHM

The Working Rule for the Sub-Trident Form to find the shortest path and the optimum solution is given by the following algorithm:

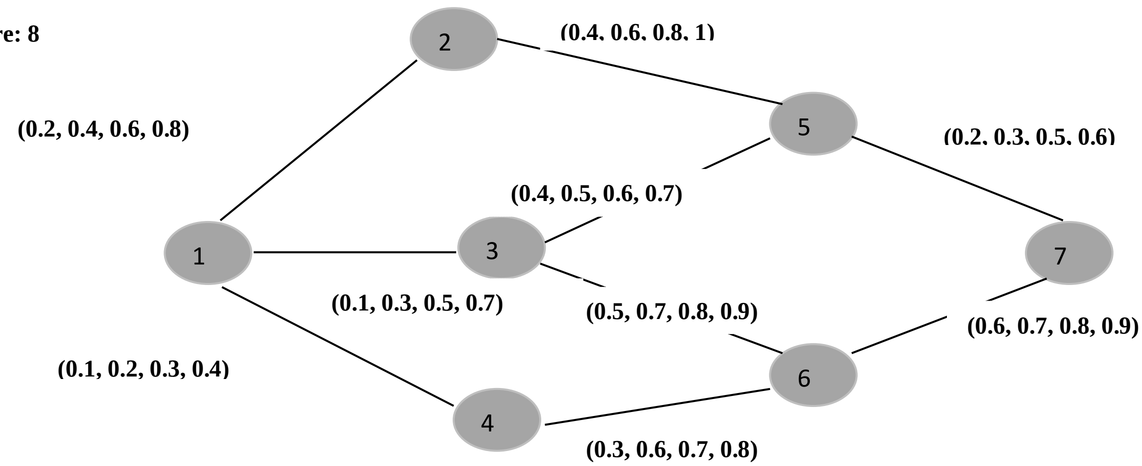
Step: 1 Input the fuzzy number as edge weight.
Step: 2 find fuzzy sub-triangular form (FST_f) of fuzzy number using Pascal's Triangle Graded Mean taken in three sides of Pascal's Triangle.
Step: 3 converting fuzzy sub-triangular form (FST_f) to Sub-Trident Form (ST_{ri}).

Step: 4 find the minimum value of the Sub-Trident Form (ST_{ri}).
Step: 5 Repeat Step: 4 for all the adjacent edges and the minimum of all adjacent edges arrive at the shortest path.
Step: 6 Optimum Solution is obtained by $optsol = \left(\sum \min ST_{ri}\right) * 100.$

ILLUSTRATIVE EXAMPLE

In order to illustrate the above procedure consider a small network shown in figure: 8 where each arc length is represented as trapezoidal fuzzy number [9]:

Figure: 8



llustrative Example

Table:

Table: 1

Path	Fuzzy Sub-Triangular Form $FST_f = (p_p, q_q, r_r)$	Sub-Trident Form (ST_{ri})	Minimum (ST_{ri})
(1,2) (1,3) **(1,4)**	(0.422,0.4,0.378) (0.267,0.3,0.278) **(0.211,0.2,0.189)**	0.7366 0.6553 **0.5847**	0.5847
(4,6)	(0.555,0.545,0.511)	0.8127	0.8127
(6,7)	(0.711,0.7001,0.689)	0.8879	0.8879
TOTAL Minimum $(ST_{ri}) = \sum \min ST_{ri}$			**2.2853**

The Minimum Value is obtained using the Sub-Trident Form in the path (1, 4). The only adjacent edge to the path (1, 4) is (4, 6) and (6, 7). Therefore the Shortest Path is $1 \rightarrow 4 \rightarrow 6 \rightarrow 7$.Suppose the node 4 is again divided into two edges then again we have to use the same Sub-Trident Form for the paths and choose the minimum value.
The Optimum Solution as the minimum cost is given by

$$optSol = \left(\sum \min ST_{ri}\right) * 100$$
$$= (2.2853) * 100$$

$= 228.53$

CONCLUSION

The aim of this paper is to find the optimum shortest path and the minimum optimum solution by using the simplest form called the Sub-Trident Form through Fuzzy Sub-Triangular Form using trapezoidal fuzzy numbers. This method is very simple while comparing to all the existing methods.

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